

The Finite Element Method Solution of an Unsteady MHD Free Convection Flow Past an Infinite Vertical Plate with Constant Suction and Heat Absorption

A Sri Sailam*

*Department of Mathematics, University College of Science, Osmania University, Hyderabad, 500007, Andhra Pradesh, India.

Abstract

The study of unsteady hydro magnetic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in the presence of constant suction and heat absorbing sinks has been made. Appropriate solutions have been derived for the velocity and temperature fields, skin friction and rate of heat transfer using Galerkin finite element method. It is observed that increase in magnetic field strength decreases the velocity of the fluid. Also the skin friction and rate of heat transfer of the conducting fluid decrease with increase in magnetic field strength.

Key Words: MHD, unsteady, free convection flow, infinite vertical plate, heat sink, constant suction, Galerkin finite element method.

I. INTRODUCTION

In recent years, the analysis of hydromagnetic convection flow involving heat and mass transfer in porous medium has attracted the attention of many scholars because of its possible applications in diverse fields of science and technology such as – soil sciences, astrophysics, geophysics, nuclear power reactors etc. In geophysics, it finds its applications in the design of MHD generators and accelerators, underground water energy storage system etc. It is worth-mentioning that MHD is now undergoing a stage of great enlargement and differentiation of subject matter. These new problems draw the attention of the researchers due to their varied significance, in liquid metals, electrolytes and ionized gases etc. The MHD in the present form is due to contributions of several notable authors like Shercliff [13], Ferraro and Plumpton [9] and Crammer and Pai[8]. Prasad *et al.*[11] discussed finite difference analysis of radiative free convection flow past an impulsively started vertical plate with variable heat and mass flux. Suneetha *et al.* [15] discussed radiation and mass transfer effects on MHD free convective Dissipative fluid in the presence of heat source/sink. Prasad *et al.* [12] studied transient radiative hydromagnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux. Singh *et al.*[14] studied the effects of permeability variation and oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate to a porous medium when the plate is subjected to a time dependent

suction velocity normal to the plate in the presence of uniform transverse magnetic field. Ahmed and Liu [1] studied the effect of heat and mass transfer mixed convective three - dimensional flow with transversely periodic suction velocity. The effect of chemical reaction magnetohydrodynamic flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption studied by Chamkha [6]. Ahmed [2] presented the effects of viscous dissipation and chemical reaction on transient free convective MHD flow over a vertical porous plate. Chen [7] discussed the combined heat and mass transfer in MHD free convection from a vertical plate with ohmic heating and viscous dissipation.

The propagation of thermal energy through mercury and electrolytic solution in the presence of external magnetic field and heat absorbing sinks has wide range of applications in chemical and aeronautical engineering, atomic propulsion, space science etc. Ahmed [3] studied free and forced convective MHD oscillatory flows over an infinite porous surface in an oscillating free stream. Transient three – dimensional flows through a porous medium with transverse permeability oscillating with time studied by Ahmed [4]. Zueco [16] discussed the numerical study of an unsteady free convective magneto hydrodynamic flow of a dissipative fluid along a vertical plate subject to a constant heat flux. Aldoss and Al – Nimir [5] has been studied the effect of the local acceleration term on the MHD transient free convection flow over a vertical plate. Muthucumaraswamy *et al.* [10] studied the heat and mass transfer effects on flow past an impulsively started vertical plate. Our objective in the present is

to study the heat transfer in mercury ($Pr = 0.025$) and electrolytic solution ($Pr = 1.0$) past an infinite porous plate with constant suction in the presence of uniform transverse magnetic field and heat sink.

II. Mathematical Analysis:

Let x' - axis be taken in the vertically upward direction along the infinite vertical plate and y' - axis normal to it. Neglecting the induced magnetic field and applying Boussinesq's approximation, the equation of the flow can be written as:

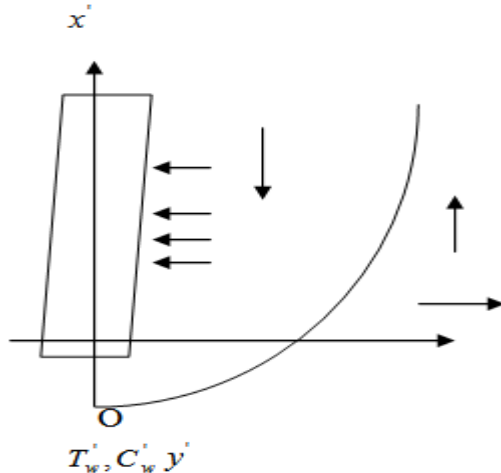


Figure 1. Physical sketch and geometry of the problem

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_o' \text{ (Constant)} \quad (1)$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T_\infty') + g\beta^*(C' - C_\infty') + v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_o'^2}{\rho} u' \quad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = K \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + S'(T' - T_\infty')$$

The boundary conditions of the problem are:

$$\left. \begin{aligned} u' = 0, v' = -v_o', T' = T_w' + \varepsilon(T_w' - T_\infty')e^{i\omega t'} \text{ at } y' = 0 \\ u' \rightarrow 0, T' \rightarrow T_\infty' \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (4)$$

Introducing the following non-dimensional variables and parameters,

$$\left. \begin{aligned} y = \frac{y' v_o'}{v}, t = \frac{t' v_o'^2}{4v}, \omega = \frac{4v\omega'}{v_o'^2}, u = \frac{u'}{v_o'}, v = \frac{\eta_o}{\rho}, \\ M = \left(\frac{\sigma B_o'^2}{\rho} \right) \frac{v}{v_o'^2}, K = \frac{K_o}{\rho C_p}, \\ T = \frac{T' - T_\infty'}{T_w' - T_\infty'}, Pr = \frac{v}{k}, Gr = \frac{v g \beta (T_w' - T_\infty')}{v_o'^3}, S = \frac{4S' v}{v_o'^2}, \\ Ec = \frac{v_o'^2}{C_p (T_w' - T_\infty')} \end{aligned} \right\} \quad (5)$$

Where

$g, \rho, v, \beta, \omega, \eta_o, k, T, T_w', T_\infty', C_p, Pr, Gr, S, K, Ec$ and M are respectively the acceleration due to gravity, density, coefficient of kinematic viscosity, volumetric coefficient of expansion for heat transfer, angular frequency, coefficient of viscosity, thermal diffusivity, temperature, temperature at the plate, temperature at infinity, specific heat at constant pressure, Prandtl number, Grashof number for heat transfer, heat source parameter, permeability parameter, Eckert number and Hartmann number.

Substituting (5) in equations (2) and (3) under boundary conditions (4), we get:

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (Gr)T + (Gc)C + \frac{\partial^2 u}{\partial y^2} - Mu \quad (6)$$

$$\frac{1}{4} \frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + \frac{1}{4} QT + (Ec) \left(\frac{\partial u}{\partial y} \right)^2 + Du \left(\frac{\partial^2 T}{\partial y^2} \right) \quad (7)$$

$$\frac{1}{4} \frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (8)$$

The corresponding boundary conditions are:

$$\left. \begin{aligned} u = 0, T = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (9)$$

III. Method of Solution:

By applying Galerkin finite element method for equation (6) over the element (e), ($y_j \leq y \leq y_k$) is:

$$\int_{y_j}^{y_k} N^T \left\{ \begin{matrix} 4 \frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} + \\ 4 \frac{\partial u^{(e)}}{\partial y} - 4Mu^{(e)} + P \end{matrix} \right\} dy = 0 \quad (9)$$

Where $P = 4(Gr)T$;

Integrating the first term in equation (9) by parts one obtains

$$N^{(e)T} \left\{ 4 \frac{\partial u^{(e)}}{\partial y} \right\}_{y_j}^{y_k} - \int_{y_j}^{y_k} \left\{ 4 \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left(\frac{\partial u^{(e)}}{\partial t} - 4 \frac{\partial u^{(e)}}{\partial y} + 4Mu^{(e)} - P \right) \right\} dy = 0 \quad (10)$$

Neglecting the first term in equation (10), one gets:

$$\int_{y_j}^{y_k} \left\{ 4 \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} + N^{(e)T} \left(\frac{\partial u^{(e)}}{\partial t} - 4 \frac{\partial u^{(e)}}{\partial y} + 4Mu^{(e)} - P \right) \right\} dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element (e)

($y_j \leq y \leq y_k$), where

$$N^{(e)} = [N_j \quad N_k], \phi^{(e)} = [u_j \quad u_k]^T \text{ and}$$

$$N_j = \frac{y_k - y}{y_k - y_j}, N_k = \frac{y - y_j}{y_k - y_j} \text{ are the basis}$$

functions. One obtains:

$$\int_{y_j}^{y_k} 4 \left\{ \begin{matrix} N'_j N'_j & N'_j N'_k \\ N'_j N'_k & N'_k N'_k \end{matrix} \right\} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy +$$

$$\int_{y_j}^{y_k} \left\{ \begin{matrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{matrix} \right\} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} dy -$$

$$4 \int_{y_j}^{y_k} \left\{ \begin{matrix} N_j N'_j & N_j N'_k \\ N'_j N_k & N'_k N_k \end{matrix} \right\} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy$$

$$+ 4M \int_{y_j}^{y_k} \left\{ \begin{matrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{matrix} \right\} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy$$

$$= P \int_{y_j}^{y_k} \begin{bmatrix} N_j \\ N_k \end{bmatrix} dy$$

Simplifying we get

$$\frac{4}{l^{(e)2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} - \frac{4}{2l^{(e)}} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{4M}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where prime and dot denotes differentiation w.r.t 'y' and time 't' respectively. Assembling the element equations for two consecutive elements $y_{i-1} \leq y \leq y_i$ and $y_i \leq y \leq y_{i+1}$ following is obtained:

$$\frac{4}{l^{(e)2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} - \frac{4}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{4M}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad (11)$$

Now put row corresponding to the node 'i' to zero, from equation (11) the difference schemes with $l^{(e)} = h$ is:

$$\frac{4}{h^2}[-u_{i-1} + 2u_i - u_{i+1}] + \frac{1}{6} \left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1} \right] - \frac{4}{2h}[-u_{i-1} + u_{i+1}] + \frac{4M}{6}[u_{i-1} + 4u_i + u_{i+1}] = P \quad (12)$$

Applying the trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^n + A_5 u_i^n + A_6 u_{i+1}^n + 12Pk \quad (13)$$

Now from equation (7) following equation is obtained:

$$G_1 T_{i-1}^{n+1} + G_2 T_i^{n+1} + G_3 T_{i+1}^{n+1} = G_4 T_{i-1}^n + G_5 T_i^n + G_6 T_{i+1}^n + 12Qk \quad (14)$$

Where $A_1 = 2 + 4Ak + 12rk - 24r$; $A_2 = 16Ak + 48r + 8$; $A_3 = 2 + 4Ak - 12rh - 24r$;

$$A_4 = 2 - 4Ak - 12rh + 24r;$$

$$A_5 = 8 - 16Ak - 48r;$$

$$A_6 = 2 - 4Ak + 12rh + 24r;$$

$$G_1 = 2(Pr) + 12rh(Pr) - S(Pr)k - 24r;$$

$$G_2 = 8(Pr) + 48r - 4S(Pr)k;$$

$$G_3 = 2(Pr) - 12rh(Pr) - 24r - S(Pr)k;$$

$$G_4 = 2(Pr) - 12rh(Pr) + 24r + S(Pr)k;$$

$$G_5 = 8(Pr) - 48r + 4S(Pr)k;$$

$$G_6 = 2(Pr) + 12rh(Pr) + 24r +$$

$$S(Pr)k; P = 4(Gr)T_i^j;$$

$$Q = 4(Pr)(Ec)k \left(\frac{\partial u_i^j}{\partial y} \right)^2$$

Here $r = \frac{k}{h^2}$ and h, k are mesh sizes along y -

direction and time - direction respectively. Index 'i' refers to space and 'j' refers to the time. In the equations (13) and (14) taking $i = 1(1) n$ and using boundary conditions (8), then the following system of equations are obtained:

$$A_i X_i = B_i \quad i = 1(1)2 \quad (15) \text{ Where}$$

A_i 's are matrices of order n and X_i, B_i 's are column matrices having n -components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity and temperature. Also, numerical solutions for these equations are obtained by C - programme. In order to prove the convergence and stability of Galerkin finite element method, the same C - programme was run with

smaller values of h and k and no significant change was observed in the values of u and T . Hence the Galerkin finite element method is stable and convergent.

IV. Skin friction and Rate of heat transfer:

The skin - friction at the plate in the dimensionless form is given by $\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0}$

And the rate of heat transfer coefficient (Nu) at the plate is $Nu = \left(\frac{\partial T}{\partial y} \right)_{y=0}$

V. Results and Discussion:

The profiles of velocity and temperature are shown in the figures 1, 2 and 3 respectively. Figure 1 exhibits the effects of Hartmann number, Prandtl number and sink - strength on the velocity profiles with other parameters are fixed.

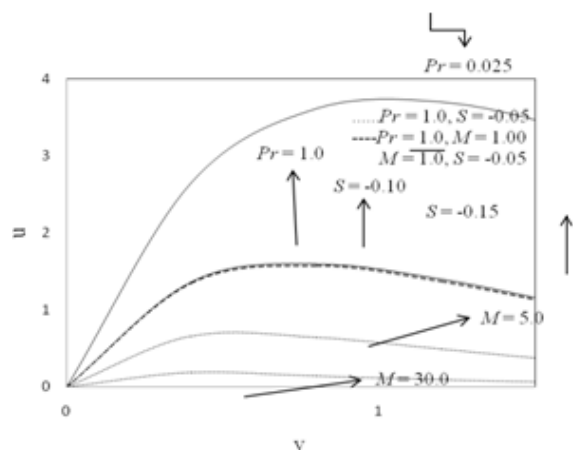


Figure 1. Effects of Pr, M and S on velocity for $Gr = 5.0, Ec = 0.001, \omega = 5.0, \varepsilon = 0.2$ and $\omega t = \pi/2$.

The effect of the Hartmann number (M) is shown in figure (1). It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Hartmann number (M) increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure (1).

And from figure 1, it is observed that the velocity is greater for mercury ($Pr = 0.025$) than that of electrolytic solution ($Pr = 1.0$). Also from figure 1 shows the effect of heat sink (S) in the case of

cooling plate ($Gr > 0$), i.e., the free convection currents convey heat away from the plate into the boundary layer. With an increase in S from -0.15 to -0.10 there is a clear increase in the velocity, i.e., the flow is accelerated. When heat is absorbed, the buoyancy force decreases, which retard the flow rate and thereby giving, rise to the increase in the velocity profiles.

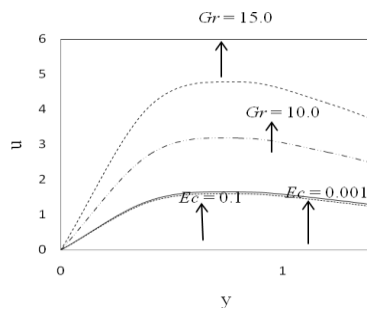


Figure 2. Effects of Gr and Ec on velocity for $Pr = 1.0$, $M = 1.0$, $\omega = 5.0$, $\varepsilon = 0.2$ and $\omega t = \pi/2$.

For various values of Grashof number (Gr) and Eckert number (Ec) the velocity profiles 'u' are plotted in figure (2). The Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. Figure 2 illustrates the effect of Eckert number Ec on velocity field under the influence of the Prandtl number Pr and the Hartmann number M . We observed an increase in Ec from 0.001 to 0.1 and that with Pr fixed either at 1.0 or at 0.025 increases in the velocity profile. However, a rise in Ec from 0.001 to 0.1 with constant Pr provides increase in the velocity profile. There is no back flow in the boundary layer for any combination of Ec and Pr .

The temperature profiles T are depicted in figure 3 for different values of heat sink parameter S , Prandtl number Pr and the Eckert number Ec . The fluid temperature attains its maximum value at the plate surface, and decreases gradually to free stream zero value far away from the plate. It is seen that the fluid temperature increases with a rise in Ec . In the present study, we restrict our attention to the positive values of Ec , which corresponds to plate cooling, i.e., loss of heat from the plate to the fluid. Also, we note that increasing Ec causes an increase in Joule heating as the magnetic field adds energy to the fluid boundary layer due to the work done in dragging the

fluid. Therefore, the fluid temperature is noticeably enhanced with an increase in S from -0.10 to -0.05 . This increase in the temperature profiles is accompanied by the simultaneous increase in the thermal boundary layer thickness.

In figure (3) we depict the effect of Prandtl number (Pr) on the temperature field. It is observed that an increase in the Prandtl number leads to decrease in the temperature field. Also, temperature field falls more rapidly for electrolytic solution and the temperature curve is exactly linear for mercury, which is more sensible towards change in temperature. From this observation it is concluded that mercury is most effective for maintaining temperature differences and can be used efficiently in the laboratory. Electrolytic solution can replace mercury, the effectiveness of maintaining temperature changes are much less than mercury. However, electrolytic solution can be better and cheap replacement for industrial purpose. This is because, either increase of kinematic viscosity or decrease of thermal conductivity leads to increase in the value of Prandtl number (Pr). Hence temperature decreases with increasing of Prandtl number (Pr). The skin friction for mercury and electrolytic solution are given in the table 1. It is noticed that the increase in magnetic field strength decreases the skin friction for both mercury and electrolytic solution. The rate of heat transfer for mercury and electrolytic solution are given in the table 2. It is observed the rate of heat transfer decreases with increase in magnetic field strength or sink strength both mercury and electrolytic solution.

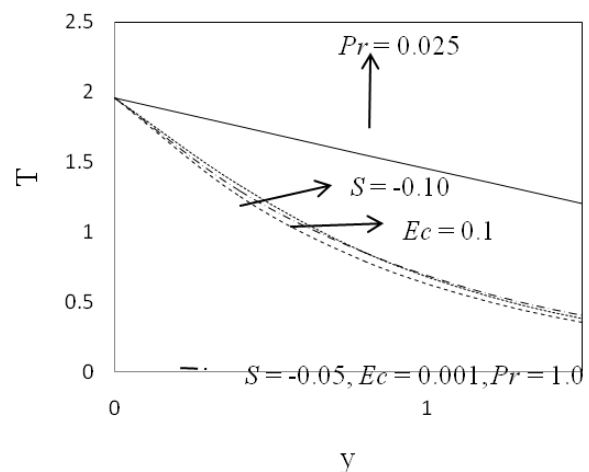


Figure 3. Effects of Pr , Ec and S on temperature for $Gr = 5.0$, $M = 1.0$, $\omega = 5.0$, $\varepsilon = 0.2$ and $\omega t = \pi/2$
 Table 1. Values of skin friction (τ) for $Gr = 5.0$, $Ec = 0.001$, $\omega = 5.0$, $\varepsilon = 0.2$ and $\omega t = \pi/2$

Pr	M	S	τ
Mercury ($Pr = 0.025$)	1.0	-0.05	7.6548
	5.0	-0.05	2.7271
	5.0	-0.10	2.7045
Electrolytic Solution ($Pr= 1.0$)	1.0	-0.05	3.0426
	5.0	-0.05	1.6584
	5.0	-0.10	1.5458

Table 2.Values of rate of heat transfer (Nu) for $Gr = 5.0$, $Ec= 0.001$, $\omega = 5.0$, $\varepsilon = 0.2$ and $\omega t = \pi/2$

Pr	M	S	Nu
Mercury ($Pr = 0.025$)	1.0	-0.05	-0.0336
	5.0	-0.05	-0.0341
	5.0	-0.10	-0.0404
Electrolytic Solution ($Pr= 1.0$)	1.0	-0.05	-0.9330
	5.0	-0.05	-1.0089
	5.0	-0.10	-1.0209

VI. CONCLUSION:

We summarize below the following results of physical interest on the velocity and temperature distribution of the flow field and also on the skin friction and rate of heat transfer at the wall.

A growing Hartmann number or Prandtl number retards the velocity of the flow field at all points.

The effect of increasing Grashof number or heat source parameter or Eckert number is to accelerate velocity of the flow field at all points.

A growing Eckert number or heat source parameter increases temperature of the flow field at all points.

The Prandtl number decreases the temperature of the flow field at all points.

A growing Hartmann number decreases the skin friction while increasing the heat source parameter increases the skin friction for both mercury and electrolytic solution.

The rate of heat transfer for both mercury and electrolytic solution is decreasing with increasing of Hartmann number and increases with increasing of heat source parameter.

Acknowledgements: The author is thankful to T Sudhakar rao professor Department of Mathematics Vasavi College of Engineering Ibrahimbagh Hyderabad,A.P., India for suggesting the problem and helpful discussions

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